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学位論文題目	Exact Order Reduction for State-Space Models of Multidimensional Systems (多次元システムの状態空間モデルの低次数化)
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論 文 内 容 要 旨

Over the last four decades, multidimensional (*n*-D) systems have received a lot of attention due to their significance in both theory and practical applications such as image processing, automatic control, circuit analysis, distributed grid sensor networks. A fundamental problem in the field of *n*-D systems is to derive a certain kind of state-space model, typically the Roesser state-space model or the Fornasini-Marchesini (F-M) (second) state-space model, from a given transfer function matrix. Moreover, the Roesser model realization is mathematically equivalent to representing a parameter-dependent matrix as a linear fractional representation (LFR) for robustness analysis and synthesis of uncertain systems. However, it has been shown that it is extremely difficult in general to obtain a minimal state-space realization for an *n*-D system. Therefore, in order to improve the accuracy and to reduce the complexity for the analysis of *n*-D systems and the LFR-based robust control of uncertain systems, it is of great importance to develop effective methods to generate realizations with lowest possible orders.

There are in general two ways to achieve this goal. One is to directly develop realization approaches that can construct a Roesser model or an F-M model from a given *n*-D transfer function or matrix with order as low as possible. And the other is to develop order reduction approaches that can further reduce, if possible, the order of a given or known *n*-D Roesser model or F-M model in an exact manner, i.e., reduce the order of a given state-space model without introducing any approximation error or changing the original input-out relation.

For establishing effective realization approaches, considerable efforts have been made and a series of significant results have been obtained. However, for the exact order reduction of *n*-D state-space model, though some preliminary results have been reported in the literature, there remain many insights and issues to be explored.

It is well known that, for the conventional 1-D system, a state-space is minimal (not reducible) if and only if it is both controllable and observable, which can be easily checked by the powerful PBH (Popov-Belevitch-Hautus) tests in terms of eigenvalues and eigenvectors. In the context of *n*-D systems, however, the reducibility problem becomes much more complicated. Completely different from the conventional 1-D counterpart, the *n*-D state-space models have more complex structures involving *n* different variables. In particular, different blocks or submatrices of the state matrix of the *n*-D Roesser model correspond to different variables, which must be treated very carefully in various scenarios. The complex nature of *n*-D systems also make the controllability and observability much more difficult, and different notions on controllability and observability of *n*-D systems have been introduced. However, these notions are not very satisfactory in the sense that a state-space model can be minimal without being controllable or observable and conversely a system can be controllable and observable without being minimal. In other words, the relationship between reducibility and controllability or observability has not yet been clearly clarified.

An approach based on the conventional 1-D reduction techniques been shown by regarding the given *n*-D system as a 1-D system in a certain variable and applying any standard 1-D reduction technique to it, and then repeating this operation successively to each of the left *n*-1 variables to obtain the final reduced-order *n*-D Roesser model. However, the effectiveness of this method is rather limited as it cannot deal with all the *n* variables simultaneously. More recently, some novel results have been obtained by restricting the paradigm to the so-called non-commuting *n*-D systems, in which, e.g., z_1z_2 is not equal to z_2z_1, for variables z_1 and z_2. By introducing the notions of structured (or generalized) Gramians, structured controllability/reachability and observability, it has been clarified that a given non-commuting *n*-D system is reducible if and only if there exists a singular structured Gramian. In principle, this approach can also be applied to a commuting system by fixing it as certain non-commuting system. However, it is easy to see that the non-commutativity is a rather strong restriction and this method cannot lead to satisfactory order reduction in general. Therefore, the essential difficulty for the reduction problem of *n*-D systems remains challenging and new approaches are highly desired.

Inspired by the fact that the PBH tests characterize the reducibility of 1-D systems in terms of eigenvalues and eigenvectors, this thesis is devoted to studying the exact order reduction of *n*-D state-space models from the point of view of eigenvalues and eigenvectors without involving the difficulties for *n*-D controllability and observability. The key idea is to introduce the notions of multiple eigenvalues and common eigenvector, by which we can successfully deal with multiple coefficient matrices related to *n*-D state-space models simultaneously and thus establish a new approach to tackle this long-standing open problem. The main results and contributions are concerned with the following four aspects:

1) First, a preliminary attempt is made for the *n*-D Roesser model based on a single eigenvalue, which only focuses on one sub-matrix corresponding one variable.
2) Second, it is shown that by using the notions of multiple eigenvalues and constrained common

eigenvector for multiple matrices, we can derive new reducibility conditions and the corresponding reduction procedures for the F-M model and the Roesser model, respectively. It will be clarified that this common eigenvector approach is applicable to a more general class of *n*-D state-space models for which the existing approaches fail to reach further order reduction. A Gröbner basis approach is also proposed to compute such a constrained common eigenvector, which leads to an alternative reducibility condition.

3) Then, the common eigenvector reduction approach is further generalized based on the notion of common invariant subspace of multiple matrices. Specifically, more general reducibility conditions and the corresponding reduction procedures are presented for the *n*-D state-space models, which includes the results based on common eigenvectors as a special case.
4) Finally, it is shown that an *n*-D Roesser model, which cannot be reduced by the proposed reduction methods and the other existing methods in the literature, may become reducible again when it is transformed into another equivalent Roesser model. Sufficient conditions and the corresponding procedure to derive such equivalent Roesser models are presented.

The thesis consists of eight chapters and is organized as follows.

1. **Introduction**
   This chapter states the background and the motivation for exact order reduction, and gives an outline of the main results of the thesis.

2. **Mathematical Preliminaries**
   In this chapter, some fundamental mathematical preparation and notions are given to make the thesis readable and self-contained.

3. **Multidimensional Systems and Exact Order Reduction**
   This chapter summarizes some basic concepts of the *n*-D systems and the existing results of exact order reduction for *n*-D systems.

4. **Reduction Based on Eigenvalue Trim**
   An eigenvalue trim approach is proposed for exact order reduction of the *n*-D Roesser model based on a single eigenvalue. Specifically, inspired by the existing notion of trim or co-trim, a new notion of eigenvalue trim, or in a dual form, eigenvalue co-trim, is introduced to the Roesser models. This result leads to new reducibility conditions and the corresponding order reduction algorithms, which can be applied to a larger class of Roesser models than the existing methods. In particular, eigenvalue trim (or eigenvalue co-trim) includes trim (or co-trim) just as a very special case, in the sense that the eigenvalue trim (or eigenvalue co-trim) implies the trim (or co-trim) but the reverse is not necessarily true. Consequently, the exact order reduction for a trim and co-trim Roesser model is impossible by the existing methods of but may still be possible by our new methods if the model is not yet eigenvalue trim or eigenvalue co-trim.

5. Reduction Based on Common Eigenvectors

A common eigenvector approach is presented to exact order reduction for *n*-D state-space models by using multiple eigenvalues and common eigenvectors. Specifically, a new notion called constrained common eigenvector is first introduced. Then, new exact reducibility conditions will be developed for the F-M model and Roesser model, respectively. It turns out that the common eigenvector approach can overcome the limitation of the existing methods on eigenvalues of one sub-matrix. These exact reducibility conditions based common eigenvectors can be viewed as a partial generalization of PBH tests for the exact reducibility of *n*-D state-space models. An effective Gröbner basis approach is proposed to compute such a constrained common eigenvector, which also leads to equivalent reducibility conditions.

6. Reduction Based on Common Invariant Subspace

In this chapter, the common eigenvector approach is extended based on a more general notion of the common invariant subspace. Then, an innovative common invariant subspace approach is derived for exact order reduction an *n*-D state-space model and it is clarified that this approach can generate a minimal state-space model of an *n*-D system in the non-commutative setting, which is from a point of view different to the methods reported in the literature. Specifically, new sufficient reducibility conditions based on common invariant subspaces are developed for the F-M model and Roesser model, respectively. It is shown that the common invariant subspace approach includes the common eigenvector approach as a special case. Based on these new reducibility conditions, new constructive reduction procedures are given for the F-M model and the Roesser model, respectively.

7. Further Exact Order Reduction

This chapter further studies the exact order reduction for the *n*-D Roesser model based on equivalence relationship. In particular, two types of transformations are firstly established to obtain equivalent Roesser models. It turns out that applying these two equivalent transformations to a minimal *n*-D Roesser model in the non-commutative setting can change the non-commutative transfer matrix of this *n*-D Roesser model and then the transformed *n*-D Roesser model may be reduced again by applying the common invariant subspace approach. Based on this fact, a novel reduction procedure is presented, which repeatedly applies the common invariant subspace approach to generate a minimal Roesser model realization in the non-commutative setting and the two equivalent transformations to obtain another Roesser model with different non-commutative transfer function matrices, such that an *n*-D Roesser model with order as low as possible can be obtained.

8. Conclusions and Future Works

This chapter summarizes the main results and provides some possible future works.

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論文審査結果要旨

複数の独立変数を持つ多次元システムの伝達関数からその状態空間モデルを求める実現問題は，分散型センサーネットワークによる環境汚染物質の検出や構造体健全性の評価，およびロバスト制御におけるシステム不確かさのモデリングなど多くの応用分野に関わる基礎課題として注目されている．しかし，従来の時間のみを独立変数とする 1 次元システムの場合と異なって，多次元状態空間実現の次数は与えられた伝達関数の次数のみならず，その構造や係数値にも依存するため，最小次数の実現を求めることは極めて困難である．従って，いかに得られた状態空間実現をさらに低次数化するかが重要な研究課題となる．本論文は，多次元状態空間モデルの低次数化問題について検討し，複数の行列に対応する多重固有値などの概念を導入することによって，共通固有ベクトルや共通不変サブ空間に基づく新しい低次数化の理論的枠組と技法を提案し，その成果をまとめたものであり，全編 8 章からなる．

第 1 章の序論に続き，第 2，3 章は，数学的な準備および多次元システムの状態空間モデルの低次数化に関する基本的な概念と従来の結果について述べている．第 4，5 章は，多次元状態方程式の係数行列と低次数化可能性との関係を解明し，多重固有値と共通固有ベクトルによる低次数化条件を与えるとともに，低次数化アルゴリズムを提案した．第 6 章は，共通固有ベクトルによる技法を複数行列の共通サブ空間へ拡張することによって，さらなる低次数化が可能であることを示し，その技法とアルゴリズムを確立した．第 7 章は，多次元状態空間モデルは提案した技法で低次数化できない場合でも等価変換でその構造を変えることによって再び低次数化できる可能性があることを解明するとともに，その判別条件と行列基本変換による等価変換の技法を提案した．第 8 章は結論である．

以上，本論文の成果は長年未解決な難問である多次元状態空間モデルの低次数化問題に対し新しい理論的枠組と解法など重要な知見を与えるものであり，システム工学の発展に寄与するところが少なくない．

よって、本論文は博士（工学）の学位論文として合格と認める．